

On Alfvén hypothesis about nuclear hydromagnetic resonances

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Abstract

The atomic nucleus capability of responding by hydromagnetic vibrations, that has been considered long ago by Hannes Alfvén, is re-examined in the context of current development of nuclear physics and pulsar astrophysics.

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1 Introduction

In the long-ago published article[1], Hannes Alfvén pointed out that electromagnetic response of an atomic nucleus should manifest features generic in hydromagnetic vibrations of an ultra fine piece of a perfectly conducting continuous medium of nuclear density with frozen-in magnetic field and made an attempt to evaluate "the order of magnitude of possible magneto-hydrodynamic (MHD) resonance frequencies"¹. In this communication we revisit this Alfvén proposition with focus on the magnitude of intranuclear magnetic field, whose presence in the nucleus volume is the chief prerequisite of sustaining MHD oscillations. In approaching this issue it seems best to start with the current understanding of "the history of matter from big bang to the present"[2], which teaches us that the nuclear material objects, both neutron stars and atomic nuclei heavier than Fe-56, are produced in the magnetic-flux-conserving core-collapse supernovae. The implosive contraction of massive main-sequence star gives birth to a neutron star - neutron-dominated mass of self-gravitating nuclear matter with frozen-in magnetic field of extremely large intensity. Since the r-process of explosive nucleosynthesis proceeds too in the presence of super strong magnetic field, it seems not inconsistent to expect that neutron-dominated mid-weight and heavy nuclei come, like neutron stars, into existence with entrapped magnetic field. In other words, the frozen-in magnetic field is the fundamental property of both neutron stars and heavy atomic nuclei.

Before embarking on theoretical underpinning for the collective model of nuclear hydro-magnetic vibrational response, we remind that basic purpose of continuum-mechanical description of nuclear giant resonances in terms of vibrational eigenstates of an ultra-fine piece of continuous nuclear matter is to gain some insight into macroscopic properties of nuclear material. The macroscopic nature of giant resonance is determined by restoring force. The position of energy centroid of a resonance in the nuclear spectrum is defined by the standard quantum-mechanical equation $E_{GR} = \hbar\omega$, where ω is the frequency of nuclear vibrations carrying information about electromagnetic and mechanical parameters of nuclear material and upon the nucleus radius $R = r_0 A^{1/3}$. The key idea is to extract the magnitude of these parameters by identifying theoretical and empirical energies of resonance under consideration. A representative example of such an approach is the macroscopic treatment of isoscalar giant resonances in terms of spheroidal and torsional modes of shear elastic vibrations of a solid sphere whose fundamental frequency reads $\omega_e = [\mu/(\rho R^2)]^{1/2}$, where μ is the shear modulus of nuclear matter. This interpretation rests on observation [20] that the energy of vibrational eigenstates, $E_e \sim \hbar\omega_e \sim A^{-1/3}$, has one and the same mass-number dependence as the empirical energy of giant isoscalar resonances $E_{GR} \sim A^{-1/3}$. The main outcome of this line of

¹By the time of publication of Alfvén work[1], all attempts to understand macroscopic properties of nuclear matter, regarded as a continuous medium, have been dominated by Gamow's idea about similarity between an atomic nucleus and a drop of liquid mercury that has been laid at the base of the nuclear liquid drop model. This similarity has been used as a guide in obtaining semi-empirical formula for nuclear binding energy[3] and, most extensively, in first macroscopic, electro-capillary, theory of nuclear fission[4]. From the history of magnetohydrodynamical investigations[5, 6, 7] it is known that the liquid mercury is the material in which hydromagnetic waves have first been discovered in widely known Lundquist's experiments. With this in mind, it seems quite plausible that all the above have led Alfvén to suggest that atomic nucleus can too respond by hydromagnetic vibrations whose excitation presumes the presence in the nucleus of frozen-in magnetic field. Together with this, it seems worth noting that while the MHD investigations have a long story, the consistent theory of vibrational MHD modes in a spherical mass has only recently been substantially developed. The extensive discussion of this theory in the context of asteroseismology of neutron stars can be found in [8-15] and in context of physics of nano-particles in [16-18].

argument consists in an assessment of shear modulus μ of nuclear material, $10^{33} < \mu < 10^{34}$ dyn cm⁻², which is of particular interest in the asteroseismology of neutron stars (e.g., [22-24] and references therein). In this context it worth mentioning that magneto-hydrodynamic theory rests on the statement that magnetic field pervading perfectly conducting medium imparts to it a supplementary portion of solid-mechanical elasticity[5, 6]. This suggests that hydromagnetic vibrations in question should have some features in common with elastic vibrations of solid sphere and, hence, manifest itself as giant resonances of isoscalar type. Adhearing to the idea that the mid-weight and heavy nuclei are produced (in r-process of explosive nucleosynthesis) with frozen-in magnetic field, we consider the nuclear MHD vibrations with focus not on the energy of hydromagnetic resonances but on the intensity of intranuclear magnetic field. Namely, having observed that the mass-number dependence of energy of hydromagnetic resonant excitations is similat to that for empirically established giant resonances we show that above line of reasoning allows one to evaluate the magnetic field magnitude.

2 Governing equations

Following a line of Alfvén's argument[1], we assume that strongly collective response of atomic nucleus (to perturbation induced by inelastically scattered electrons or elastically scatted photons) is dominated by hydromagnetic vibrations. The relevant to this case MHD equations can be conveniently written in the form[5]

$$\rho \delta \dot{\mathbf{v}} = \frac{1}{c} [\delta \mathbf{j} \times \mathbf{B}], \quad \delta \mathbf{j} = \frac{c}{4\pi} [\nabla \times \delta \mathbf{B}], \quad (1)$$

$$\delta \dot{\mathbf{B}} = \nabla \times [\delta \mathbf{v} \times \mathbf{B}], \quad \nabla \cdot \delta \mathbf{v} = 0 \quad (2)$$

These equations describe Lorenz-force-driven oscillations of velocity $\delta \mathbf{v}$ of material flow coupled with fluctuations of magnetic field $\delta \mathbf{B}$ about immobile equilibrium state of incompressible and perfectly conducting continuous medium of density ρ pervaded by magnetic field \mathbf{B} . Taking into account that $\delta \mathbf{v} = \dot{\mathbf{u}}$ where \mathbf{u} is the field of material displacement (which is the basic variable of solid-mechanical theory of elasticity), the coupled equations (1) and (2) can be reduced to only one equation²

$$\rho \ddot{\mathbf{u}} = \frac{1}{4\pi} [\nabla \times [\nabla \times [\mathbf{u} \times \mathbf{B}]] \times \mathbf{B}]. \quad (3)$$

In approximation of node-free vibrations, widely used in macroscopic models of collective nuclear dynamics, the frequency of Alfvén hydromagnetic modes can be computed by the energy method which rests on integral equation of energy balance

$$\frac{\partial}{\partial t} \int \frac{\rho \dot{\mathbf{u}}^2}{2} d\mathcal{V} = \frac{-1}{4\pi} \int [\mathbf{B} \times [\nabla \times [\nabla \times [\mathbf{u} \times \mathbf{B}]]] \cdot \dot{\mathbf{u}} d\mathcal{V} \quad (4)$$

² The above mentioned analogy between oscillatory behavior of perfectly conducting medium pervaded by magnetic field (magneto-active plasma) and elastic solid, regarded as a material continuum, is strengthened by the following tensor representation of the last equation $\rho \ddot{u}_i = \nabla_k \delta M_{ik}$, where $\delta M_{ik} = (1/4\pi)[B_i \delta B_k + B_k \delta B_i - B_j \delta B_j \delta_{ik}]$ is the Maxwellian tensor of magnetic field stresses with $\delta B_i = \nabla_k [u_i B_k - u_k B_i]$. This form is identical in appearance to canonical equation of solid-mechanics $\rho \ddot{u}_i = \nabla_k \sigma_{ik}$, where $\sigma_{ik} = 2\mu u_{ik} + [\kappa - (2/3)\mu] u_{jj} \delta_{ik}$ is the Hookean tensor of mechanical stresses and $u_{ik} = (1/2)[\nabla_i u_k + \nabla_k u_i]$ is the tensor of shear deformations in an isotropic elastic continuous matter with shear modulus μ and bulk modulus κ (having physical dimension of pressure).

In this method, the bulk density ρ and the shape of frozen-in magnetic field \mathbf{B} are regarded as known functions of position, $\rho(r) = \rho f(r)$ and $\mathbf{B}(\mathbf{r}) = B \mathbf{b}(\mathbf{r})$, where $\rho = \text{constant}$ is the density in the nucleus center and dimensionless scalar function $f(r)$ describes the density profile, by $B = \text{constant}$ is denoted the magnetic field intensity and $\mathbf{b}(\mathbf{r})$ stands for the dimensionless vector-function of spatial distribution of the field over the nucleus volume. Substitution in the latter equation of the following separable representation of fluctuating material displacements

$$\mathbf{u}(\mathbf{r}, t) = \mathbf{a}(\mathbf{r}) \alpha(t), \quad (5)$$

where $\mathbf{a}(\mathbf{r})$ is the time-independent field of instantaneous displacements, leads to equation for amplitude $\alpha(t)$ describing harmonic vibrations

$$\frac{d\mathcal{H}_A}{dt} = 0, \quad \mathcal{H}_A = \frac{\mathcal{M}\dot{\alpha}^2(t)}{2} + \frac{\mathcal{K}\alpha^2(t)}{2}, \quad (6)$$

$$\mathcal{M}\ddot{\alpha}(t) + \mathcal{K}\alpha(t) = 0, \quad \mathcal{M} = \rho m, \quad \mathcal{K} = \frac{B^2}{4\pi} k, \quad (7)$$

$$m = \int f(r) \mathbf{a}(\mathbf{r}) \cdot \mathbf{a}(\mathbf{r}) d\mathcal{V}, \quad (8)$$

$$k = \int \mathbf{a}(\mathbf{r}) \cdot [\mathbf{b}(\mathbf{r}) \times [\nabla \times [\nabla \times [\mathbf{a}(\mathbf{r}) \times \mathbf{b}(\mathbf{r})]]]] d\mathcal{V}. \quad (9)$$

The general analytic expression for the spectrum of discrete frequencies of MHD oscillations, $\omega_\ell(MHD)$, can be represented as follows

$$\omega_\ell(MHD) = \sqrt{\frac{\mathcal{K}}{\mathcal{M}}} = \omega_A(B) s_\ell, \quad \omega_A(B) = \frac{v_A}{R}, \quad v_A = \frac{B}{\sqrt{4\pi\rho}} \quad (10)$$

where $\omega_A(B)$ stands for the Alfvén frequency which is the natural unit of frequency of MHD oscillations depending only on the field strength B and s_ℓ is numerical spectral factor depending of multipole degree ℓ of hydromagnetic oscillations. As illustrative example relevant to the subject of this work, we present the result of calculations with frozen-in magnetic field of an axisymmetric configuration, pictured in Fig.1, whose spherical components are

$$\mathbf{b} = \left[b_r = 0, b_\theta = 0, b_\phi(r, \theta) = \frac{(R^2 - r^2 \sin^2 \theta)^{1/2}}{R} \right]. \quad (11)$$

The frequency spectrum of MHD vibrations with the node-free irrotational field of instantaneous displacements (as is the case of nuclear giant resonances of electric type)

$$\mathbf{a} = A_\ell \nabla [r^\ell P_\ell(\theta)] \quad (12)$$

(where $P_\ell(\theta)$ being Legendre polynomial of multipole degree ℓ) is given by

$$\omega_\ell(MHD) = \omega_A(B) s_\ell, \quad s_\ell = \left[\frac{(2\ell + 1)(\ell - 1)}{(2\ell - 1)} \right]^{1/2}. \quad (13)$$

The basic frequency of Alfvén oscillations ω_A of a spherical mass $M = (4\pi/3)\rho R^3$ can be represented in the following equivalent form

$$\omega_A(B) = B \sqrt{\frac{R}{3M}} \quad (14)$$

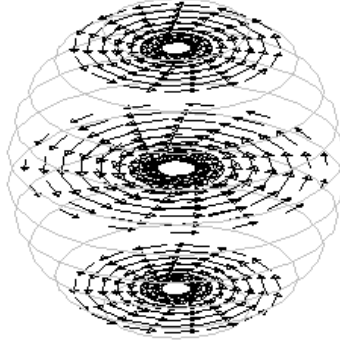


Figure 1: An illustrative example of lines of frozen-in magnetic field having axisymmetric pure toroidal configuration: $\mathbf{B}_t = [B_r = 0, B_\theta = 0, b_\phi(r, \theta) = (B/R)[R^2 - r^2 \sin^2 \theta]^{1/2}]$ with $B = \text{constant}$.

In what follows we use the standard parametrization of the nucleus mass $M = m A$ (where $m = 1.66 \times 10^{-24}$ g is the atomic unit of the nucleon mass) and radius $R = r_0 A^{1/3}$ (with $r_0 = 1.2 \times 10^{-13}$ cm). On account of this one finds that mass-number dependence of energies (in MeV) of expected hydromagnetic resonant modes is given by

$$E_\ell(MHD) = \hbar \omega_\ell(MHD) = \kappa_\ell B A^{-1/3}, \quad \kappa_\ell = \hbar \left[\frac{r_0}{3m} \right]^{1/2} s_\ell, \quad (15)$$

$$\kappa_\ell = 1.6 \times 10^{-22} \times s_\ell, \quad [2 < s_\ell < 6, \quad 2 < \ell < 4].$$

Equation (15) exhibits the fact that the nuclear hydromagnetic resonances, if exist, are characterized by one and the same dependence of energy centroids upon mass number A as empirically established giant resonances

$$E_{GR}(\ell) = C_\ell A^{-1/3} = \text{constant}. \quad (16)$$

This suggests, if some of detected giant resonances are of predominantly hydromagnetic nature, then from identification of above theoretical and experimental estimates, $E_\ell(MHD)A^{1/3} = E_{GR}(\ell)A^{1/3}$, one can evaluate the magnitude of nuclear internal magnetic field B . For the giant resonant modes lying in the energy interval

$$30 < E_{GR}A^{1/3} < 90, \quad (5 \leq E_{GR} \leq 15) \text{ [MeV]}, \quad (17)$$

$$4.8 \times 10^{-5} < E_{GR}A^{1/3} < 1.4 \times 10^{-4} \text{ [erg]} \quad (18)$$

we obtain

$$3.0 \times 10^{17} \leq B \leq 9.0 \times 10^{17} \text{ G}. \quad (19)$$

Remarkably, that according to QCD estimates of radius distribution of magnetic moment in a nucleon (whose origin is attributed to persistent Fermi-motion of quarks) falls in the

range $0.3 < R_N < 0.6$ fm. Taking into account that nuclear magneton, $\mu_N = 5.05 \times 10^{-24}$ erg/G, it is easy to see that the intensity of dipole magnetic field $B = 2\mu_N/R_N^3$ on the magnetic poles of sphere of above radius (spherical region occupied by quark matter) is ranged in the interval: $5.0 \times 10^{16} - 5.0 \times 10^{17}$ G. Similar argument has been used in paper[24], devoted to the possibility of ferromagnetic state of superdense matter. The above estimates give an idea about magnetic field intensity in the quark matter which is expected to exist in deep cores of neutrons stars[25]. In this latter context also noteworthy that current investigations on search for the chiral magnetic effects lead to the conclusion that magnetic fields of above strength should be generated in heavy-ion collisions at intermediate energies[26, 27]. It seems interesting to note that the magnetic field energy stored in spherical volume of nuclear radius, $R = r_0 A^{1/3}$, is proportional to the mass number A , namely: $W_B \sim B^2 R^3 \sim A$, as is the case of volume-energy-term in semi-empirical formula for the nuclear binding energy. This suggests that the volume term of nuclear binding energy may be of magnetic origin, that is, due to the energy of huge magnetic field stored in the nucleus on the stage of explosive nucleosynthesis. In this connection it is appropriate to note that synthesis of chemical elements in the presence of a super strong magnetic fields of magnetars has recently been studied in [28-31] with remarkable conclusion that the fields of order of $B \sim 10^{17}$ G can substantially affect both the r-process of neutron capture and formation of shell nuclear structure, that is, magic nuclei with enhanced stability. Finally, it may be worth mentioning the well-known in astrophysics argument[32] regarding the effect of strong internal magnetic field on the star shape: the prevailed poloidal magnetic field leads to the oblate deformation of the star shape, whereas the toroidal field leads to prolate deformation³. From this perspective, it is not implausible to expect, therefore, that it is the super strong internal magnetic field plays decisive part in the formation of equilibrium shapes of nuclei heavier than Fe-56.

3 Summary

As a development of Alfvén hypothesis about the atomic nucleus capability of responding by magneto-hydrodynamic vibrations, we have set up a collective model providing theoretical basis for computing energies of nuclear hydromagnetic resonances. The central to this model, which is appropriate to nuclei heavier than Fe-56, is the intranuclear magnetic field. This field is considered as being frozen-in the mid-weight and heavy nuclei on the stage of their formation in the r-process of explosive nucleosynthesis. The model predicts that the energy of nuclear hydromagnetic resonances is a linear function of internal magnetic field. The mass-number dependence of energy has one and the same shape as that for typical giant resonances. Based on this and assuming that some of observed giant resonances are predominantly of hydromagnetic nature we found that the intensity of intranuclear magnetic field falls in the realm of magnetic fields of magnetars.

³It may be worth noting that this paper of Sweet [32] is the one in which the conservation of magnetic flux density in the process of the main-sequence star formation from gravitationally contracting gas-dust interstellar medium has been discussed for the first time. As is commonly known today, Ginzburg and Woltjer were the first to suggest that super strong magnetic fields of neutron stars can too be explained as due to the magnetic flux conservation in process of gravitation collapse of massive main-sequence stars.

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